# On–line Call Control in Cellular Networks*

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**Abstract**

In this paper we study an important communication problem in cellular networks that utilize Frequency Division Multiplexing (FDM) technology. In such networks, many users within the same geographical region can communicate simultaneously with other users of the network using distinct frequencies. Since the spectrum of available frequencies is limited, the important engineering problem to be solved is to efficiently allocate frequencies in such a way that the number of users served is maximized (call control problem).

We study two call control algorithms (the greedy and the "classify and randomly select" algorithm) and present results on their performance using competitive analysis. We also prove lower bounds for deterministic and randomized call control algorithms and propose directions for future research on the problem.

## 1 Introduction

In the area of mobile computing, which combines wireless and high speed networking technologies, rapid technological progress has been made. It is expected that in the near future, mobile users will have access to a wide variety of services available over communication networks.

An architectural approach widely common for wireless networks is the following. A geographical area in which communication takes place is divided into regions. Each region is the calling area of a base station. Base stations are connected via a high speed network. The topology of the high speed network is not of interest; the only requirement is that it is connected. When a mobile user A wishes to communicate with some other user B, a path must be established between the base stations of the regions in which the users A and B are located. Then communication is performed in three steps: (a) wireless communication between A and its base station, (b) communication between the base stations, and (c) wireless communication between B and its base station. Thus, the transmission of a message from A to B first takes place between A and its base station, the base station of A sends the message to the base station of B which will transmit it to B. At least one base station is involved in the communication even if both A and B locate in the same region.

Many users of the same region can communicate simultaneously with other users of the network. This can be achieved via frequency division multiplexing (FDM). The base station is responsible

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for allocating distinct frequencies from the available spectrum to users so that signal interference is avoided both within the same region and adjacent regions.

Since the spectrum of available frequencies is limited, the important engineering problem to be solved is to efficiently allocate frequencies in such a way that the number of users served is maximized.

Note that we ignore the communication problems arising in the high speed network that interconnects the base stations. This can be efficiently solved using protocols that are supported by modern high speed networks (e.g. ATM).

The network topology adopted is a finite portion of the infinite triangular lattice. This results from the uniform distribution of base stations within the network, as wells as from the fact that the calling area of a base station is a circle which, for simplicity reasons, is idealized as a regular hexagon. Associated with the cellular network is an *interference graph* $G$ that reflects possible signal interference. Vertices correspond to cells and an edge $(u, v)$ exists in the graph if and only if the cells corresponding to $u$ and $v$ are adjacent. Due to geometry, we call this graph a hexagon graph (see figure 1). If the above assumptions (uniform distribution of base stations and equivalence of transmitters) do not hold, arbitrary interference graphs can be used to model the underlying network. At the rest of this paper, we use the term cellular network especially for networks with hexagon interference graph, like the one depicted in figure 1.

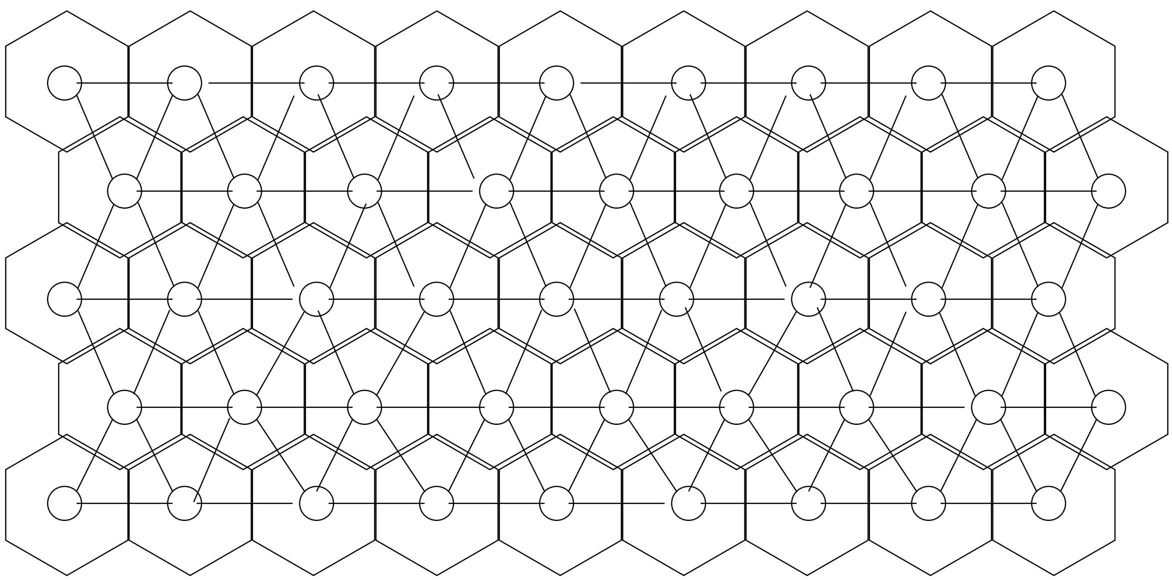

Figure 1: A cellular network and the corresponding interference (hexagon) graph

Given users that with to communicate, two problems are of main interest:

- The *frequency allocation* problem is to assign frequencies to the users so that signal interference is avoided, minimizing the total number of frequencies used.

- The *call control* (or call admission) problem on a network that supports a spectrum of $w$ available frequencies, is to assign frequencies to users so that signal interference is avoided, maximizing the number of users served.

There is a primal–dual relation between the frequency allocation and the call control problem.

Frequency allocation on cellular networks has recently received much attention [7, 5, 3], while the call control problem in networks with arbitrary intereference graphs is studied in [8].

In this paper, we address the on–line version of the call control problem. We study intuitive on–line algorithms (like the greedy algorithm and the "classify and randomly select" algorithm) and present results on their performance using competitive analysis. We also prove lower bounds on the competitiveness of any on–line call control algorithm (possibly randomized).

The rest of the paper is structured as follows. We refine our model and give some preliminary definitions about competitive analysis in section 2. We study the performance of our algorithms in section 3. The lower bounds are presented in section 4. We conclude with open problems in section 5.

## 2 Preliminaries

We assume that calls corresponding to users that wish to communicate appear in the cells of the network in an on–line manner. When a call arrives, an on–line algorithm decides either to accept the call assigning a frequency to it, or to reject it. Once a call is accepted, it cannot be rejected (preempted). Furthermore, the frequency assigned to the call cannot be changed in the future. We assume that all calls have infinite duration; this assumption is equivalent to considering calls of the same duration.

Competitive analysis [9] has been used for evaluating the performance of on–line algorithms for various problems. In our setting, given a sequence of calls, the performance of an on–line algorithm $A$ is compared to the performance of the optimal algorithm $A_{OPT}$.

Let $B_A(\sigma)$ be the benefit of the on–line algorithm $A$ on the sequence of calls $\sigma$, i.e. the number of calls of $\sigma$ accepted by $A$ and $B_{OPT}(\sigma)$ the benefit of the optimal algorithm $A_{OPT}$.

If $A$ is a deterministic algorithm, we define its competitive ratio $c$ as

$$c = \max_{\sigma} \frac{B_{OPT}(\sigma)}{B_A(\sigma)},$$

where the maximum is taken over all possible sequences of calls.

If $A$ is a randomized algorithm, we define its competitive ratio $c$ as

$$c = \max_{\sigma} \frac{B_{OPT}(\sigma)}{\mathcal{E}[B_A(\sigma)]},$$

where $\mathcal{E}[B_A(\sigma)]$ is the expectation of the number of calls accepted by $A$, and the maximum is taken over all possible sequences of calls.

Usually, we compare the performance of deterministic algorithms against *off–line adversaries*, i.e. adversaries that have knowledge of the behaviour of the deterministic algorithm in advance. In the case of randomized algorithms, we consider *oblivious adversaries* whose knowledge is limited to the probability distribution of the random choices of the randomized algorithm.

## 3 Upper bounds

An intuitive algorithm proposed for the call control problem in many different contexts [8], is the greedy algorithm which can be described as follows. Consider a network that supports $w$ frequencies. Wlog we assume that frequencies are positive integers $1, 2, ..., w$. Given a sequence of calls $\sigma = r_1, r_2, \ldots$, the greedy algorithm accepts $r_i$ by assigning it the smallest frequency available; i.e. the smallest frequency that has not been assigned to calls located in cells adjacent to the cell of $r_i$. If no frequency is available, the greedy algorithm rejects $r_i$.

Pantziou et al. in [8] study the competitiveness of the greedy algorithm in arbitrary interference graphs and prove that its competitive ratio is $\Delta + 1$, where $\Delta$ is the maximum degree of the interference graph. We extend their techniques and prove the following theorem.

**Theorem 1** *Let $G = (V, E)$ be an interference graph, $v$ a vertex of $G$, and $\Gamma_v$ the maximum independent set in the neighborhood of $v$. The greedy algorithm is $\frac{1}{1-e^{-\frac{1}{\gamma}}}$–competitive against an off–line adversary, where*

$$\gamma = \max_{v \in V} |\Gamma_v|.$$

We first prove the following.

**Lemma 1** *The greedy algorithm is $\gamma$ competitive against an off–line adversary in networks that support one frequency.*

**Proof:** Let $F_A$ be the set of calls accepted by the greedy algorithm and $F_{OPT}$ the set of calls accepted by the optimal algorithm.

The number of calls rejected by $A$ because of the calls in $F_A \backslash F_{OPT}$ is at most $\gamma |F_A \backslash F_{OPT}$. Indeed, the best the optimal algorithm could do is to reject a call $r_i \in F_A \backslash F_{OPT}$ in a cell $c_i$ accepted by the greedy algorithm, and accept at most $\gamma$ calls that appear in cells adjacent to $c_i$.

Since the network supports only one frequency, no call in $F_{OPT} \cap F_A$ can cause the rejection of any other call in $F_{OPT}$, because the calls in $F_{OPT}$ appear in distinct not adjacent cells. Thus,

$$|F_{OPT} \backslash F_A| \leq \gamma |F_A \backslash F_{OPT}| \Rightarrow$$

$$|F_{OPT} \backslash F_A| + |F_{OPT} \cap F_A| \leq \gamma |F_A \backslash F_{OPT}| + |F_{OPT} \cap F_A| \Rightarrow$$

$$|F_{OPT}| \leq \gamma |F_A|.$$

The lemma follows. ■

Let $A$ be a greedy algorithm for one frequency. In general, for networks with multiple frequencies, a greedy algorithm $A'$ can be formally described as follows.

**Greedy algorithm $A'$ for $w$ frequencies**

1. for $i = 1$ to $w$ do:
2.     run $A$ on $\sigma$ and output the subset $\sigma_i$ of calls accepted by $A$.
3.     assign frequency $i$ to calls in $\sigma_i$.
4.     $\sigma = \sigma \backslash \sigma_i$.
5.     if $\sigma = \emptyset$ exit.

Note that the above description is equivalent to the one given at the beginning of this section. The competitiveness of on–line algorithms for many frequencies that can be expressed as an iterative execution of an algorithm for one frequency has been studied in the context of optical networks in [1, 10]. The next lemma, proved in [10], holds in our setting as well.

**Lemma 2** *If $A$ has competitive ratio $c$ against against an off–line adversary, then $A'$ has competitive ratio at most*

$$\frac{1}{1 - e^{-\frac{1}{c}}}$$

*against an off–line adversary.*

Theorem 1 follows by lemmas 1 and 2. It can be easily seen that

$$\frac{1}{1 - e^{-\frac{1}{\gamma}}} < \Delta + 1.$$

For cellular networks, where the interference graph is a hexagon graph, it is $\gamma = 3$, and theorem 1 yields the following corollary.

**Corollary 1** *The greedy algorithm is 3.53–competitive against an off–line adversary when applied to cellular networks.*

We now define a randomized algorithm $A_{CRS}$ that follows the "classify and randomly select" paradigm [1, 8]. Given the interference graph $G$ of the network, consider a proper coloring of the vertices of $G$; equivalently, a coloring of cells so that adjacent cells are assigned different colors. In particular, the hexagon interference graph of a cellular network can be colored with $\chi(G) = 3$ colors. According to this coloring, the algorithm $A_{CRS}$ classifies the calls into three classes according to the color assigned to the cell the call is contained in. Initially, $A_{CRS}$ selects uniformly at random one of these classes, and considers only calls that belong to this class, rejecting all other calls.

**Theorem 2 ([1, 8])** *The algorithm $A_{CRS}$ is 3–competitive against an oblivious adversary.*

## 4 Lower bounds

In this section we prove our lower bounds. We consider networks that support one frequency; our bounds can be trivially extended for networks that support multiple frequencies. We first prove that the greedy algorithm is not far from optimal within the class of deterministic algorithms by presenting the following lower bound.

**Theorem 3** *No deterministic algorithm can be better than 3–competitive against an off–line adversary.*

**Proof:** Consider a cellular network that supports one frequency, a deterministic call–control algorithm $A$ and the following off–line adversary $\mathcal{ADV}$. Let $C$ be a set of cells such that the distance between any two cells in $C$ is at least 4. In an odd step $2i+1$, the adversary produces the call $r_i$ in cell $c_i \in C$; in the even steps the adversary produces:

- no call, if $A$ rejected $r_i$, or
- three calls in distinct not adjacent cells which are adjacent to $c_i$, if $A$ accepted $r_i$.

It is $B_{OPT}(\sigma) \geq 3B_A(\sigma)$ and the theorem follows. ∎

Obviously, the best performance is achieved if $A$ accepts any call that is located at a cell not adjacent to a cell containing a call previously accepted; this is exactly the definition of the greedy algorithm for one frequency.

In the following we prove a lower bound on the competitive ratio of randomized algorithms. We prove the lower bound in a similar manner to the game–theoretic technique proposed by Yao [11] for proving lower bounds on the running time of randomized algorithms (see also [6]). We use the following lemma.

**Lemma 3 (Yao's Minimax Principle [6])** *Given a probability distribution $\mathcal{P}$ over sequences of calls $\sigma$, denote by $\mathcal{E}_{\mathcal{P}}[B_A(\sigma)]$ and $\mathcal{E}_{\mathcal{P}}[B_{OPT}(\sigma)]$ the expected benefit of a deterministic algorithm $A$ and the optimal off–line algorithm on sequences of calls generated according to $\mathcal{P}$. Define the competitiveness of $A$ under $\mathcal{P}$, $c_A^{\mathcal{P}}$ to be such that*

$$c_A^{\mathcal{P}} = \frac{\mathcal{E}_{\mathcal{P}}[B_{OPT}(\sigma)]}{\mathcal{E}_{\mathcal{P}}[B_A(\sigma)]}.$$

*Let $A_R$ be a randomized algorithm. Then, the competitiveness of $A$ under $\mathcal{P}$ is a lower bound on the competitive ratio of $A_R$ against an oblivious adversary, i.e.*

$$c_A^{\mathcal{P}} \leq c_{A_R}.$$

**Theorem 4** *No randomized call–control algorithm can be better than $5/3$–competitive against an oblivious adversary when applied to a cellular network.*

**Proof:** We will prove that there exists an adversary that selects calls according to a probability distribution $\mathcal{P}$ in such way that no deterministic algorithm can be better than $5/3$–competitive under $\mathcal{P}$, even if it knows the probability distribution $\mathcal{P}$ in advance.

Consider the following adversary $\mathcal{ADV}$ that produces input for a call control algorithm. Let $C$ be a set of cells such that the distance between any two cells in $C$ is at least 4. In an odd step $2i+1$, the adversary produces a call $r_i$ in cell $c_i \in C$; in the even steps the adversary produces:

- either three calls in distinct not adjacent cells which are adjacent to the cell $c_i$, with probability $1/3$, or
- no call, with probability $2/3$.

Assume that the adversary runs for a period of $2t$ steps.

Consider a call control algorithm $A$ that runs on the calls produced by $\mathcal{ADV}$. Let $k$ be the number of odd steps in which $A$ accepts (the number of odd steps in which $A$ rejects is $t-k$).

We denote by $\mathcal{X}$, the random variable indicating the number of odd steps in which $A$ rejects and $\mathcal{ADV}$ produces three calls at the next step. By linearity of expectation, the expectation of $\mathcal{X}$ when calls are produced according to $\mathcal{P}$ is

$$\mathcal{E}_{\mathcal{P}}[\mathcal{X}] = \frac{t-k}{3}.$$

The best any algorithm can do, is to reject the call in odd steps if $\mathcal{ADV}$ is going to produce calls in the next even step, or accept the call produced in the odd step otherwise, and to accept any call produced in an even step. Denote by $\mathcal{Y}$ the random variable indicating the number of even steps in which $\mathcal{ADV}$ produces no calls. It is

$$\mathcal{E}_{\mathcal{P}}[\mathcal{Y}] = \frac{2t}{3}.$$

Thus

$$\mathcal{E}_{\mathcal{P}}[B_{OPT}(\sigma)] = \mathcal{E}_{\mathcal{P}}[\mathcal{Y}] + 3(t - \mathcal{E}_{\mathcal{P}}[\mathcal{Y}]) = \frac{5t}{3},$$

and

$$c_A^{\mathcal{P}} \geq 5/3.$$

By lemma 3, we obtain that this is a lower bound on the competitive ratio of any randomized algorithm against an oblivious adversary. ■

## 5 Future work

Closing the gap between our lower and upper bounds is an interesting open problem. Due to the lower bound for deterministic algorithms, investigation of randomized competitive algorithms is necessary. The "classify and randomly select" is not satisfactory not only because we conjecture that its competitiveness is far from optimality, but also for another important reason. Consider a network that supports one frequency, a "classify and randomly select" algorithm $A$, and a sequence $\sigma$ of calls that appear in distinct cells belonging to the same color class. Obviously, the optimal algorithm accepts all these calls, while, with probability $2/3$, algorithm $A$ gains no benefit.

It is desirable not only to design randomized algorithms that achieve an expected benefit which is close to that of the optimal algorithm; further research on the problem should be directed to call–control algorithms whose benefit is sharply concentrated around its expectation. Such results have been recently presented in the context of optical networks [4].

Moreover, although our assumption for calls with infinite durations captures the scenario where calls utilize the network for almost equal ammount of time, this may not be the case in practice. Extending our model so that it considers the duration of calls as a parameter, and designing algorithms in this context, is another interesting challenge.

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