11 Brain-Twisting Paradoxes

Paradoxes have been around since the time of Ancient Greeks & the credit of popularizing them goes to recent logicians. Using logic you can usually find a fatal flaw in the paradox which shows why the seemingly impossible is either possible or the entire paradox is built on flawed thinking. Can you all work out the problems in each of the 11 paradoxes shown here? If you do, post your solutions or the fallacies in the comments.

11
The Omnipotence Paradox

The paradox states that if the being can perform such actions, then it can limit its own ability to perform actions and hence it cannot perform all actions, yet, on the other hand, if it cannot limit its own actions, then that is—straight off—something it cannot do. This seems to imply that an omnipotent being's ability to limit itself necessarily means that it will, indeed, limit itself. This paradox is often formulated in terms of the God of the Abrahamic religions, though this is not a requirement. One version of the omnipotence paradox is the so-called paradox of the stone: "Could an omnipotent being create a stone so heavy that even that being could not lift it?" If so, then it seems that the being could cease to be omnipotent; if not, it seems that the being was not omnipotent to begin with. An answer to the paradox is that having a weakness, such as a stone he cannot lift, does not fall under omnipotence, since the definition of omnipotence implies having no weaknesses.

10
The Sorites' Paradox

The paradox goes as follows: consider a heap of sand from which grains are individually removed. One might construct the argument, using premises, as follows:

1,000,000 grains of sand is a heap of sand. (Premise 1)

A heap of sand minus one grain is still a heap. (Premise 2)

Repeated applications of Premise 2 (each time starting with one less grain), eventually forces one to accept the conclusion that a heap may be composed of just one grain of sand.

On the face of it, there are some ways to avoid this conclusion. One may object to the first premise by denying 1,000,000 grains of sand makes a heap. But 1,000,000 is just an arbitrarily large number, and the argument will go through with any such number. So the response must deny outright that there are such things as heaps. Peter Unger defends this solution. Alternatively, one may object to the second premise by stating that it is not true for all collections of grains that removing one grain from it still makes a heap. Or one may accept the conclusion by insisting that a heap of sand can be composed of just one grain.

9
The Interesting number paradox

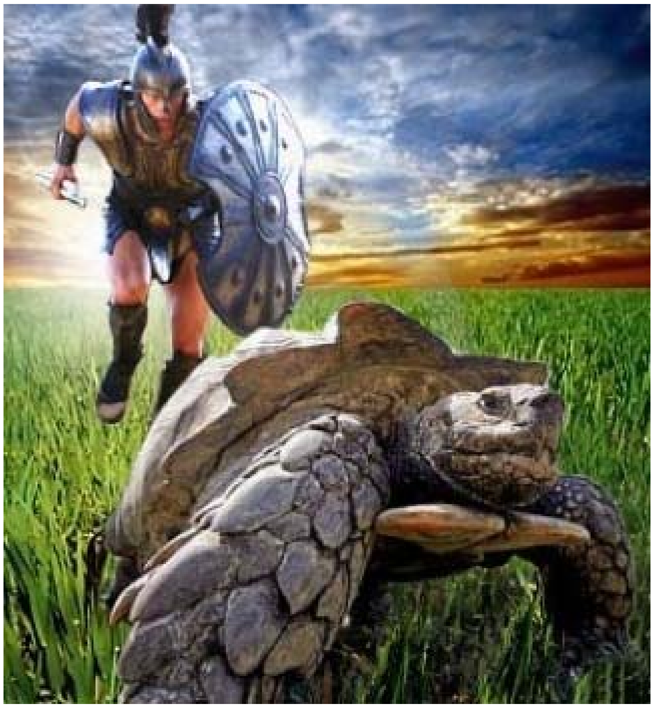
Claim: There is no such thing as an uninteresting natural number.

Proof by Contradiction: Assume that you have a non-empty set of natural numbers that are not interesting. Due to the well-ordered property of the natural numbers, there must be some smallest number in the set of not interesting numbers. Being the smallest number of a set one might consider not interesting makes that number interesting. Since the numbers in this set were defined as not interesting, we have reached a contradiction because this smallest number cannot be both interesting and uninteresting. Therefore the set of uninteresting numbers must be empty, proving there is no such thing as an uninteresting number.

8
The arrow paradox

In the arrow paradox, Zeno states that for motion to be occurring, an object must change the position which it occupies. He gives an example of an arrow in flight. He states that in any one instant of time, for the arrow to be moving it must either move to where it is, or it must move to where it is not. It cannot move to where it is not, because this is a single instant, and it cannot move to where it is because it is already there. In other words, in any instant of time there is no motion occurring, because an instant is a snapshot. Therefore, if it cannot move in a single instant it cannot move in any instant, making any motion impossible. This paradox is also known as the fletcher's paradox—a fletcher being a maker of arrows. Whereas the first two paradoxes presented divide space, this paradox starts by dividing time – and not into segments, but into points.

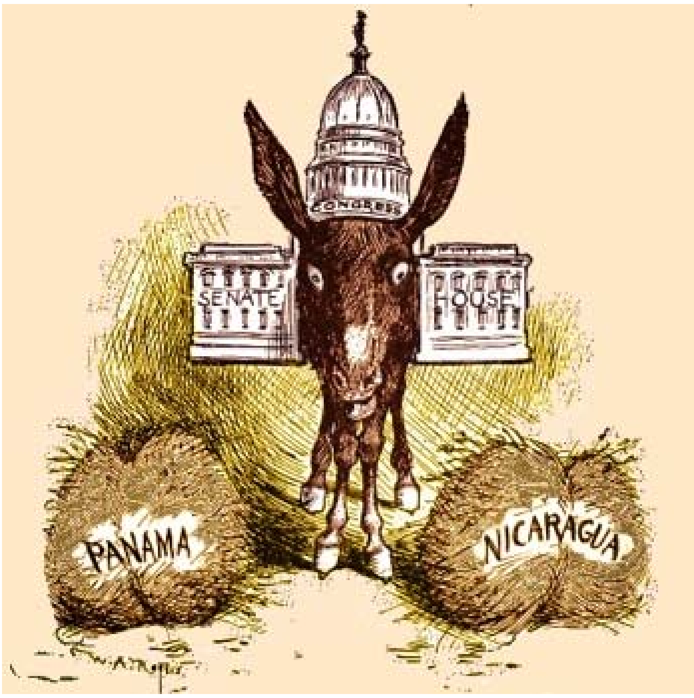
7
Achilles & the tortoise paradox



In the paradox of Achilles and the Tortoise, Achilles is in a footrace with the tortoise. Achilles allows the tortoise a head start of 100 feet. If we suppose that each racer starts running at some constant speed (one very fast and one very slow), then after some finite time, Achilles will have run 100 feet, bringing him to the tortoise's starting point. During this time, the tortoise has run a much shorter distance, say, 10 feet. It will then take Achilles some further time to run that distance, by which time the tortoise will have advanced farther; and then more time still to reach this third point, while the tortoise moves ahead. Thus, whenever Achilles reaches somewhere the tortoise has been, he still has farther to go. Therefore, because there are an infinite number of points Achilles must reach where the tortoise has already been, he can never overtake the tortoise. Of course, simple experience tells us that Achilles will be able to overtake the tortoise, which is why this is a paradox.

[JFrater: I will point out the problem with this paradox to give you all an idea of how the others might be wrong: in physical reality it is impossible to transverse the infinite - how can you get from one point in infinity to another without crossing an infinity of points? You can't - thus it is impossible. But in mathematics it is not. This paradox shows us how mathematics may appear to prove something - but in reality, it fails. So the problem with this paradox is that it is applying mathematical rules to a non-mathematical situation. This makes it invalid.]

6
The Buridan's ass paradox



This is a figurative description of a man of indecision. It refers to a paradoxical situation wherein an ass, placed exactly in the middle between two stacks of hay of equal size and quality, will starve to death since it cannot make any rational decision to start eating one rather than the other. The paradox is named after the 14th century French philosopher Jean Buridan. The paradox was not originated by Buridan himself. It is first found in Aristotle's De Caelo, where Aristotle mentions an example of a man who remains unmoved because he is as hungry as he is thirsty and is positioned exactly between food and drink. Later writers satirised this view in terms of an ass who, confronted by two equally desirable and accessible bales of hay, must necessarily starve while pondering a decision.

5
The unexpected hanging paradox

A judge tells a condemned prisoner that he will be hanged at noon on one weekday in the following week, but that the execution will be a surprise to the prisoner. He will not know the day of the hanging until the executioner knocks on his cell door at noon that day. Having reflected on his sentence, the prisoner draws the conclusion that he will escape from the hanging. His reasoning is in several parts. He begins by concluding that the "surprise hanging" can't be on a Friday, as if he hasn't been hanged by Thursday, there is only one day left – and so it won't be a surprise if he's hanged on a Friday. Since the judge's sentence stipulated that the hanging would be a surprise to him, he concludes it cannot occur on Friday. He then reasons that the surprise hanging cannot be on Thursday either, because Friday has already been eliminated and if he hasn't been hanged by Wednesday night, the hanging must occur on Thursday, making a Thursday hanging not a surprise either. By similar reasoning he concludes that the hanging can also not occur on Wednesday, Tuesday or Monday. Joyfully he retires to his cell confident that the hanging will not occur at all. The next week, the executioner knocks on the prisoner's door at noon on Wednesday — which, despite all the above, will still be an utter surprise to him. Everything the judge said has come true.

4
The barber's Paradox

Suppose there is a town with just one male barber; and that every man in the town keeps himself clean-shaven: some by shaving themselves, some by attending the barber. It seems reasonable to imagine that the barber obeys the following rule: He shaves all and only those men in town who do not shave themselves.

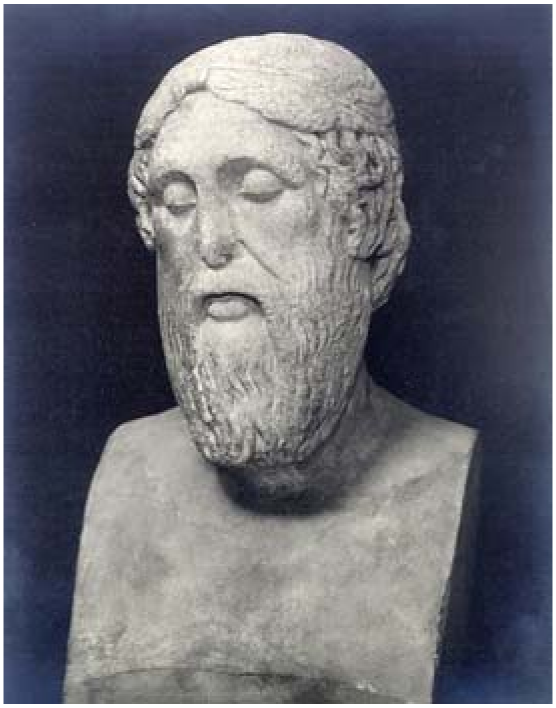
Under this scenario, we can ask the following question: Does the barber shave himself?

Asking this, however, we discover that the situation presented is in fact impossible:

- If the barber does not shave himself, he must abide by the rule and shave himself.

- If he does shave himself, according to the rule he will not shave himself

3
Epimenides' Paradox



This paradox arises from the statement in which Epimenides, against the general sentiment of Crete, proposed that Zeus was immortal, as in the following poem:

They fashioned a tomb for thee, O holy and high one The Cretans, always liars, evil beasts, idle bellies! But thou art not dead: thou livest and abidest forever, For in thee we live and move and have our being.

He was, however, unaware that, by calling all Cretens liars, he had, unintentionally, called himself one, even though what he 'meant' was all Cretens except himself. Thus arises the paradox that if all Cretens are liars, he is also one, & if he is a liar, then all Cretens are truthful. So, if all Cretens are truthful, then he himself is speaking the truth & if he is speaking the truth, all Cretens are liars. Thus continues the infinite regression.

2
The paradox of the court

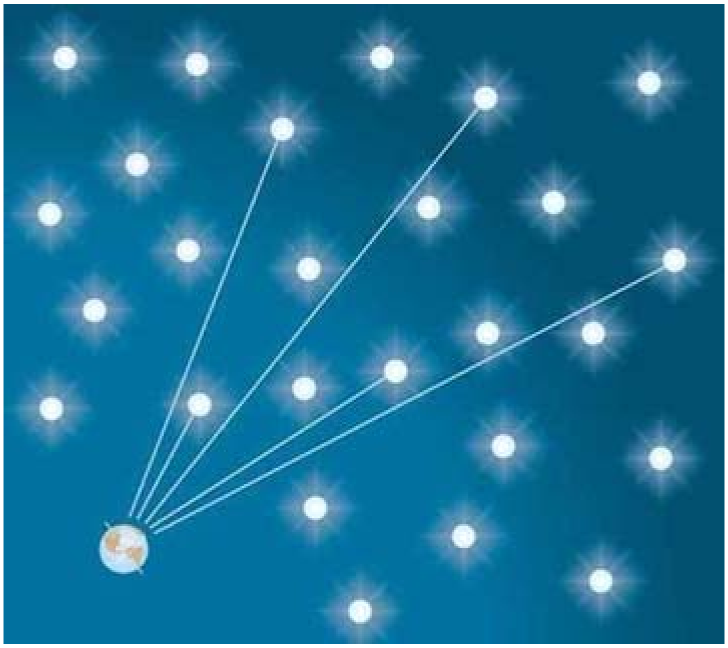
The Paradox of the Court is a very old problem in logic stemming from ancient Greece. It is said that the famous sophist Protagoras took on a pupil, Euathlus, on the understanding that the student pay Protagoras for his instruction after he had won his first case (in some versions: if and only if Euathlus wins his first court case). Some accounts claim that Protagoras demanded his money as soon as Euathlus completed his education; others say that Protagoras waited until it was obvious that Euathlus was making no effort to take on clients and still others assert that Euathlus made a genuine attempt but that no clients ever came. In any case, Protagoras decided to sue Euathlus for the amount owed. Protagoras argued that if he won the case he would be paid his money. If Euathlus won the case, Protagoras would still be paid according to the original contract, because Euathlus would have won his first case.

Euathlus, however, claimed that if he won then by the court's decision he would not have to pay Protagoras. If on the other hand Protagoras won then Euathlus would still not have won a case and therefore not be obliged to pay. The question is: which of the two men is in the right?

1
The unstoppable force paradox

The Irresistible force paradox, also the unstoppable force paradox, is a classic paradox formulated as "What happens when an irresistible force meets an immovable object?" The paradox should be understood as an exercise in logic, not as the postulation of a possible reality. According to modern scientific understanding, no force is completely irresistible, and there are no immovable objects and cannot be any, as even a minuscule force will cause a slight acceleration on an object of any mass. An immovable object would have to have an inertia that was infinite and therefore infinite mass. Such an object would collapse under its own gravity and create a singularity. An unstoppable force would require infinite energy, which does not exist in a finite universe.

Bonus
Olbers' Paradox



In astrophysics and physical cosmology, Olbers' paradox is the argument that the darkness of the night sky conflicts with the assumption of an infinite and eternal static universe. It is one of the pieces of evidence for a non-static universe such as the current Big Bang model. The argument is also referred to as the "dark night sky paradox" The paradox states that at any angle from the earth the sight line will end at the surface of a star. To understand this we compare it to standing in a forest of white trees. If at any point the vision of the observer ended at the surface of a tree, wouldn't the observer only see white? This contradicts the darkness of the night sky and leads many to wonder why we do not see only light from stars in the night sky.